Black hole entropy as entanglement entropy: a holographic derivation

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Abstract: We study the possibility that black hole entropy be identified as entropy of entanglement across the horizon of the vacuum of a quantum field in the presence of the black hole. We argue that a recent proposal for computing entanglement entropy using AdS/CFT holography implies that black hole entropy can be exactly equated with entanglement entropy. The implementation of entanglement entropy in this context solves all the problems (such as cutoff dependence and the species problem) typically associated with this identification.

Contents

1. Introduction

The vacuum of a quantum field in equilibrium with a black hole contains correlations between points inside and outside the horizon that are responsible for the thermal behavior detected by outside observers [1, 2]. It is then plausible that black hole entropy receives a contribution, possibly accounting for all of it, from the entropy of the density matrix of a quantum field obtained by tracing over degrees of freedom beyond the horizon, i.e., the entropy of entanglement across the horizon [3] (see [4] for a discussion and references to the subject). The leading contribution to the entropy of entanglement across a geometric boundary does indeed come out naturally as proportional to the area of the boundary [3, 5], and so it seems to reproduce a main feature of the Bekenstein-Hawking formula[1]

$$S_{BH} = \frac{\mathcal{A}_h}{4G\hbar} \,. \tag{1.1}$$

However, attempts to push this identification further have typically faced a number of difficulties:

- The entropy of entanglement is divergent due to correlations of modes with arbitrarily short wavelengths close to the dividing boundary [3, 6, 5]. If this is regularized with an ultraviolet cutoff λ_{UV} then the resulting entropy is strongly cutoff-dependent, $S \sim \mathcal{A}/\lambda_{UV}^{D-2}$.

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[1]It will be useful to keep $\hbar$ explicit in our equations.

- The species problem: if one wants to fully account for black hole entropy as entanglement entropy, then the number of fields contributing to the entanglement entropy appears to need a miraculous adjustment (compounded by the cutoff dependence) in order to exactly match the Bekenstein-Hawking formula (1.1).

- The gravitational backreaction of the quantum fields is typically neglected. This backreaction may not only distort the black hole area as well as change the field's contribution to entanglement. It will also generate higher-derivative corrections to the gravitational action that will result in contributions to black hole entropy that are not proportional to the area.

While quantum contributions to black hole entropy of the form $\mathcal{A}_h/\lambda_{UV}^{D-2}$ may be absorbed in a renormalization of Newton's constant [7], it is much less clear whether all of the black hole entropy can be understood in this way, through some 'induced gravity' mechanism [4, 8].

Here we discuss how the application of a recently proposed prescription to compute entanglement entropy [9], based on AdS/CFT holography, implies that black hole entropy can be exactly identified with the entropy of entanglement of quantum fields across the black hole horizon. In the setup we devise, all of the above difficulties are solved or circumvented in a simple manner.

The gist of our derivation is to have a black hole localized at the boundary of AdS, and not at its center — the latter leads to a different notion of entanglement entropy in AdS/CFT [10, 11] that we will discuss below, before the conclusions. The idea of having a horizon at the boundary to study the entanglement of the CFT was pioneered by [12], who applied it to the problem of understanding the entropy of deSitter space. By restricting themselves to a very specific calculable case, namely deSitter in $1 + 1$ dimensions, the authors of ref. [12] were able to perform a field-theoretic calculation of the entanglement entropy and compare it to the area formula. Here we take a different route, and compute instead the entanglement entropy using the proposal of [9]. This method allows us to treat much more general cases, and in particular to include actual black holes in our arguments, but, as we discuss in the concluding section, being a bulk-based calculation, it does not directly clarify the microscopic origin of the entropy.

2. Holographic calculation of entanglement entropy

In ref. [9] Ryu and Takayanagi propose, and give evidence for, the following way to compute the entropy of entanglement for a (conformal) quantum field theory that admits a gravitational dual. Say that the D-dimensional field theory is defined on the conformal timelike boundary ∂M of a $(D+1)$-dimensional bulk spacetime M (typically an AdS geometry), and that we wish to compute the entropy of entanglement of a subsystem $A \subset \partial M$ bounded by a $(D-2)$-dimensional spatial surface ∂A. Build then a $(D-1)$-dimensional static minimal surface γ_A in M whose boundary in ∂M is ∂A. The entanglement entropy in A is then

$$S_A = \frac{\mathcal{A}(\gamma_A)}{4G_{D+1}\hbar},$$ (2.1)

where $\mathcal{A}(\gamma_A)$ is the area of the minimal surface γ_A and G_{D+1} is Newton's constant, both defined in the bulk spacetime M. The divergences in the entanglement entropy mentioned in the introduction are reflected here in the generically infinite area of γ_A as it extends to asymptotic infinity. So far the proposal has been formulated only for static surfaces, but an extension to stationary surfaces, as required when rotation is present, might be possible. In this paper, though, we will confine ourselves to static situations.

Although clearly inspired by the Bekenstein-Hawking formula (1.1), this prescription is different from it in that (2.1) is in general intended to apply to spacelike surfaces that are not horizons, in fact the boundary ∂A is typically an artificial division. However, given the similarity between (1.1) and (2.1) it seems plausible that a connection between the two should exist. Our aim is to describe a concrete set up where the identification of black hole entropy with entanglement entropy, computed via AdS/CFT, becomes manifest.

3. Set up

Ref. [9] applied the proposal (2.1) to cases where the bulk spacetime is essentially empty or contains a black hole at its center (not at the boundary), which corresponds to a thermal state of the quantum conformal field theory. We want, instead, that the spacetime in which the field theory lives contains a black hole, i.e., we need a black hole at the boundary.

Another ingredient in our construction is the introduction of an ultraviolet cutoff for the conformal field theory, which is needed in order to render finite the entanglement entropy. This is naturally incorporated in AdS/CFT by cutting off from the bulk geometry the spacetime extending from a given timelike surface, "the brane", out to infinity. As a result of this, not only the D-dimensional CFT dual to the bulk is cut off in the UV, but also bulk gravity now contains a normalizable graviton mode localized on the brane [13] that gives rise, in the dual field theory, to D-dimensional gravity coupled to the cutoff CFT. Thus, the classical $(D+1)$-dimensional bulk theory containing such a brane provides a dual description of the dynamics on the boundary brane satisfying the D-dimensional equations [14]

$$G_{\mu\nu} = 8\pi G_D \langle T_{\mu\nu} \rangle \,, \tag{3.1}$$

where the Einstein tensor $G_{\mu\nu}$ and the Newton constant G_D are those induced on the boundary brane from the bulk, G_D being related to the bulk constant G_{D+1} and AdS radius R by

$$G_D = \frac{D-2}{R} G_{D+1} \,. \tag{3.2}$$

$\langle T_{\mu\nu} \rangle$ is the expectation value of the renormalized stress-energy tensor of the CFT, including all leading (planar) corrections in a $1/N$ expansion. Hence the boundary theory accounts for the exact gravitational backreaction of the quantum CFT at planar level. The cutoff length of the CFT is equal to the AdS radius,
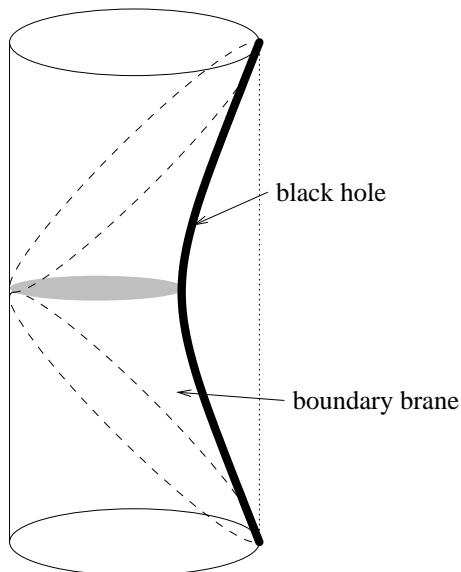
$$\lambda_{UV} = R \,. \tag{3.3}$$

Figure 1: Conformal diagram of the spacetime for a black hole on a brane in AdS. On a spatial slice at constant time (shaded) the horizon of the black hole is a minimal surface.

This is the version of AdS/CFT duality that applies to the Randall-Sundrum model with a single Planck brane (RS2). Exactly what is the dual CFT depends on the specific construction at hand: we shall only need to assume that it is possible to make sense of the duality at least as long as bulk quantum corrections are negligible. The decomposition of degrees of freedom in terms of a CFT coupled to D-dimensional gravity is only sensible as an effective theory at energies well below the cutoff. In this regime we do not need to deal with the specific nature of the brane, which becomes relevant only if we need to know the ultraviolet completion of the theory.

We shall now focus on $D = 3$, since in this case there exists an explicit exact bulk solution for a black hole localized on a RS2 three-brane in AdS$_4$ [15] (see figure 1). The metric induced on the brane

$$ds^2|_{\text{brane}} = -\left(1 - \frac{r_0}{r}\right) dt^2 + \frac{dr}{1 - r_0/r} + r^2 d\phi^2 \tag{3.4}$$

contains a black hole horizon, as we required. It can be seen that the angle ϕ has periodicity less than 2π so there is a conical deficit that extends out to infinity [15]. This geometry satisfies the three-dimensional Einstein equations with stress-energy tensor

$$T^\mu{}_\nu = \frac{1}{16\pi G_3} \frac{r_0}{r^3} \, \text{diag}(1, 1, -2) \,. \tag{3.5}$$

According to the discussion above, this stress-energy tensor must be interpreted as the renormalized $\langle T^\mu{}_\nu \rangle$ of the CFT.

It is well known that there are no asymptotically flat black holes in three-dimensional vacuum gravity, but instead point masses give rise to locally flat conical spacetimes. Refs. [16, 17] argued that, from the point of view of the dual CFT+gravity 3D theory,

the horizon in (3.4) is the result of backreaction of the quantum conformal field on a cone through eq. (3.1). Ref. [17] provided strong evidence for this effect. In fact, if we write the horizon radius r_0 in terms of quantities of the dual 3D theory, including the number of degrees of freedom g_* of the CFT ,

$$g_* \sim \frac{R}{\hbar G_3} = \frac{R^2}{\hbar G_4} , \qquad (3.6)$$

one finds $r_0 = \hbar g_* G_3 \, f(G_3 M)$, indicating the quantum origin of the horizon. Since we assume that we are in a regime where g_* is very large (in order to suppress bulk quantum gravity effects), the horizon can have macroscopic size much larger than the 3D Planck length $\hbar G_3$ or the CFT cutoff.

Moreover, when expressed in terms of 3D quantities the entropy of the black hole is proportional to $\hbar^0 g_*$. This strongly suggests that the entropy of this black hole may have its origin in quantum degrees of freedom of the CFT, possibly from entanglement of the vacuum across the horizon. Since we do have the explicit bulk dual of the black hole+CFT configuration (3.4), (3.5), it seems natural to apply the formula (2.1) to the calculation of this entanglement entropy.

Before proceeding to this analysis, let us first identify the entropy of the black hole (3.4). The metric (3.4) is only the brane-section of a four-dimensional bulk solution, given in [15], with a black hole that extends into the bulk. Ref. [15] argued that the entropy of the black hole is given by the Bekenstein-Hawking formula applied to the two-dimensional area $\mathcal{A}_h$ of the black hole horizon *in the bulk*,[2]

$$S_{BH} = \frac{\mathcal{A}_h}{4 G_4 \hbar} = \frac{\pi R}{2 G_3 \hbar} \frac{x^2}{1 + 3x/2} , \qquad (3.7)$$

where x is an auxiliary variable that satisfies $x^2(1+x) = r_0^2/R^2$, and in the last equality we have expressed the result in terms of G_3 using (3.2). This is *not* the same as the Bekenstein-Hawking entropy associated to the one-dimensional length of the horizon circumference on the brane, $\mathcal{C}_h$,

$$S_\circ = \frac{\mathcal{C}_h}{4 G_3 \hbar} = \frac{\pi R}{2 G_3 \hbar} \frac{x(x+1)}{1 + 3x/2} . \qquad (3.8)$$

The difference between (3.7) and (3.8) is attributed to the expected corrections to the effective action induced by renormalization of the CFT. However, the two entropies become approximately the same in the limit $r_0 \gg R$ (i.e., large x) in which the black hole is much larger than the cutoff length of the CFT, in accordance with the expectation that higher-derivative corrections to the action should be suppressed in this limit. These corrections, conveniently expressed in an expansion in $R/\mathcal{C}_h$, are

$$S_{BH} = S_\circ \left(1 - \frac{4\pi}{3} \frac{R}{\mathcal{C}_h} + \cdots \right) . \qquad (3.9)$$

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[2]This S_{BH} differs from [15] by a factor of 2, since we are taking the brane to be single-sided instead of double-sided.

4. Black hole entropy equals entanglement entropy

Following the prescription (2.1), in order to compute the entanglement entropy of the CFT in the region A outside the black hole horizon, we must find a minimal surface γ_A in a spatial slice of the bulk geometry whose boundary on the brane ∂A coincides with the horizon of the black hole on the brane.

Since we are assuming staticity, this is actually very easy. On a time-symmetric spatial slice, an apparent horizon (a surface where the expansion of outgoing null geodesics vanishes) is a minimal surface.[3] So in a static spacetime, in which the event horizon is also an apparent horizon, if we take a spatial slice at constant Killing time the horizon will appear as a minimal surface. Readers unfamiliar with this property of apparent horizons in time-symmetric initial data sets may find this puzzling, since it seems to go against the flat space intuition that a sphere at constant radius is obviously not a minimal surface. That it is nevertheless true for a black hole, can be easily visualized by considering the section of the black hole on a spatial slice that goes through the Einstein-Rosen bridge.

Therefore we identify the required γ_A as the (spatial section at constant Killing time of the) horizon in the bulk. It follows at once that the entanglement entropy (2.1) is the same as the entropy of the black hole localized on the brane (3.7),

$$S_{BH} = S_A(\partial A = \text{horizon}|_{\text{brane}}) \tag{4.1}$$

(even if none of them is given by the area of the horizon on the brane!).

The proof of this result is so simple that it immediately begs the question of how it avoids the difficulties mentioned at the beginning of the paper — concerns that this might be a trivial circular argument will be addressed in the discussion at the end.

The first two issues — cutoff dependence and species problem — are basically dealt with together (similar points were made in [12]). The RS2-AdS/CFT construction that we have used comes naturally equipped with a UV cutoff for the CFT, (3.3), that is at the same time responsible for the strength of the gravitational coupling induced on the brane G_3 through eq. (3.2). This same parameter fixes the number of species of the dual CFT, in such a way that the effective G_3 is correctly reproduced. In more detail, when working at length scales much larger than the cutoff R, we expect the contribution of each field to the entanglement entropy to be of the order of

$$S_{(\text{single field})} \sim \frac{\mathcal{C}_h}{R}, \tag{4.2}$$

so for a number g_* of fields the total entanglement entropy is

$$S \sim g_* \frac{\mathcal{C}_h}{R} \sim \frac{\mathcal{C}_h}{G_3 \hbar}, \tag{4.3}$$

where in the last line we have used (3.6) (the numerical factors come out correctly once we fix g_* so that a thermal state of the CFT reproduces the entropy of a black hole in AdS$_4$: this assumption follows from the fact that we are always doing calculations in the

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[3]More properly, an extremal surface. Under suitable usual conditions it will also be minimal.
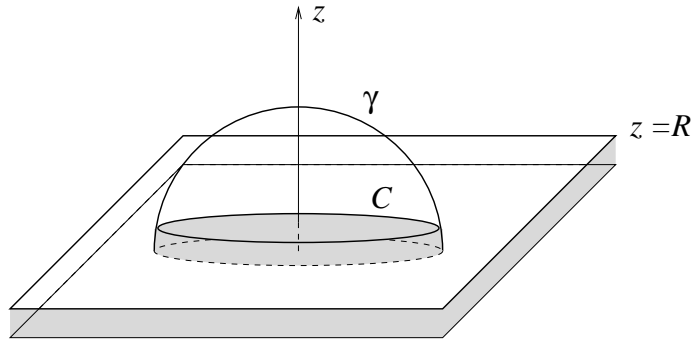
Figure 2: Minimal surface γ computing the entanglement across a circle C on a flat plane. A spatial section of AdS is represented as the Poincare upper-half-volume. The shaded region $0 \leq z < R$ is cut off in the RS2 construction. The minimal surface is a hemisphere, of which only the portion above $z = R$ is relevant. The length of the circle C is fixed to be $\mathcal{C}_h$. The proper distance along z is $R \log(z/R)$, so when $R/\mathcal{C}_h \ll 1$ the hemisphere is actually pancaked along the brane.

same side of the AdS/CFT duality). This adjustment of the number of degrees of freedom is similar to the argument in the induced-gravity scenario, in which the effective Newton constant G_3 also depends on the number of species [4].

Note that we have only included the entanglement of the CFT as the primary contribution to the entropy. Quantum fluctuations of the graviton on the brane (which result in quantum fuzzines in the position of the horizon) will also be entangled across the horizon. However, in the large N (i.e., large g_*) limit the number of modes of the graviton is $O(1)$, compared to $g_* \gg 1$ for the CFT. The graviton contribution is then subleading, and only becomes relevant when quantum gravity in the bulk becomes important. Observe that in the regime in which we work, the cutoff length R of the CFT is much larger than either the three- or four-dimensional Planck lengths, so the entanglement entropy of each individual mode is much smaller than the Bekenstein-Hawking entropy, $S_{(\text{single field})} \ll S_\circ \simeq S_{BH}$.

Finally, the gravitational backreaction of the CFT is incorporated through the equation (3.1) of RS2-AdS/CFT duality. The fact that the surface γ_A is the same as the spatial section of the bulk horizon guarantees that the black hole entropy will be the same as the entanglement entropy also after including corrections that change the entropy away from the Bekenstein-Hawking entropy on the brane $S_\circ$.

5. Comparison to entanglement on a circle in flat space

It is instructive to compare the entanglement across the black hole horizon to the entanglement on a circle (since we are in two spatial dimensions) of the same length in flat space. This is analogous to the calculations in [3, 5], but now we use the holographic method (2.1) to compute the entanglement entropy.

Therefore, we take AdS$_4$ with a RS2 two-brane that is flat (and without any conical deficit),

$$ds^2 = \frac{R^2}{z^2}(-dt^2 + dz^2 + dr^2 + r^2 d\phi^2), \qquad z \geq R, \tag{5.1}$$

and consider as our entanglement boundary a circle C on the brane with length equal to $\mathcal{C}_h$ (given by (3.8)), $r = \mathcal{C}_h/2\pi$. We must find the area of a minimal surface in the bulk, γ, that is bounded by the circle C. This calculation has actually been done in [9], and results in an entanglement entropy equal to

$$
\begin{aligned}
S_C &= \frac{\mathcal{C}_h}{4G_3\hbar} \left(\sqrt{1 + \left(\frac{2\pi R}{\mathcal{C}_h}\right)^2} - \frac{2\pi R}{\mathcal{C}_h} \right) \\
&= S_\circ \left(1 - \frac{2\pi R}{\mathcal{C}_h} + \cdots \right),
\end{aligned}
\tag{5.2}
$$

where we have expanded for small $R/\mathcal{C}_h$ to compare with (3.9). We see that the leading term agrees in both cases. The reason that the agreement includes the precise factors, without any ambiguities from cutoff dependence, is the same as explained in the preceding section. Geometrically, it is easy to see (figure 2) that the minimal surface γ is 'pancaked' along the brane in a manner very similar to that of horizons of large black holes in RS2 [15], so their total bulk area is $\simeq \mathcal{C}_h R$ in both cases. The factor of R (the cutoff length) in this bulk area is then absorbed into G_3 in the computation of the entropy.

The subleading corrections to the entropy in (5.2), which are again interpreted as due to higher-derivative terms in the action, differ by a factor $2/3$ from the corrections in (3.9). This numerical discrepancy is not surprising, but it is interesting to see that the sign of the corrections is the same in both cases.

In the opposite limit of small black holes, $R/\mathcal{C}_h \gg 1$, the entropy S_C can only account for half of S_{BH} (one half being the ratio between the areas of a disk and a hemisphere of the same radius), again not unexpected in a regime dominated by higher-derivative operators. Note however that the identity (4.1) between S_{BH} and S_A on the horizon holds irrespective of the size of the black hole.

6. Generalization to higher dimensions

No exact solution for a static black hole with regular horizon on a RS2 brane in $\mathrm{AdS}_{D>4}$ is known. Refs. [16, 17] argue that indeed no such solution should exist, since the backreaction of the quantum fields should make the black hole shrink as a consequence of Hawking evaporation. However, the possibility that strong coupling effects of the CFT complicate the picture seems to leave some room for skeptics [18]. Within this paper we can remain agnostic as to the final resolution of this problem, and discuss both possibilities. The conclusion that entanglement entropy is the primary source for black hole entropy is not essentially affected by this issue.

If static regular black holes on RS2 branes in $\mathrm{AdS}_{D>4}$ exist, it is clear that, since in any dimension the horizon is a minimal surface on a slice at constant time, then the black hole entropy is in general equal to the holographic definition of the entanglement entropy of the CFT, (4.1). A holographic calculation of the entanglement entropy on a sphere in flat space of the same area as the black hole horizon, will again reproduce precisely the leading contribution $S_\circ$ to the black hole entropy.

If, on the contrary, black holes on the brane are necessarily time-dependent due to Hawking emission of dual CFT radiation, then the situation becomes more complicated since the outgoing radiation is also expected to be correlated with the black hole state. Still, the time evolution may be slow enough to allow for an approximate holographic definition of entanglement entropy. In the regime of validity of the effective CFT+gravity theory, in which the black hole is much larger than the cutoff size, the evaporation rate should in fact be small. So in this regime we also expect that entanglement entropy, possibly computed from a minimal surface very close to the horizon at some constant-time slice, accounts for most of the black hole entropy. However, we will not attempt here to make any of these notions more precise. It should be very interesting to study how the information about correlations escapes out of the black hole.

Finally, we mention that, if in addition to the ultraviolet cutoff, an infrared cutoff brane is introduced, then it is possible to have regular black holes in the bulk in the form of black strings extending between the UV and IR branes (the distance between them must be closer than R in order to avoid dynamical instabilities). In this case, our general holographic argument applies again to prove that black hole entropy is exactly equal to entanglement entropy.

7. Connection to another notion of entanglement within AdS/CFT

Our approach to relate black hole entropy to entanglement entropy using AdS/CFT holography looks conceptually very different from another approach discussed recently [10] (see also [11]). The notion of entanglement considered in [10] involves a configuration where a black hole is *at the center of AdS*, instead of at its boundary. The maximally extended spacetime (the 'eternal black hole') contains two causally disconnected asymptotically AdS regions, 1 and 2, each with its own causal boundary. So there are two copies of the dual CFT — CFT_1 and CFT_2—, and the total Hilbert space is $\mathcal{H} = \mathcal{H}_1 \times \mathcal{H}_2$. This is a thermofield double: the total state of the CFT is a pure state, but if we trace over the degrees of freedom in $\mathcal{H}_2$ then we obtain a thermal density matrix for the state in CFT_1. In contrast to the situation we have been discussing, these two copies of the CFT do not share any common geometric dividing surface: the boundary where the CFT is defined does not contain a black hole.

It is however possible to relate this notion of entanglement to the one we have studied here. To this effect, we follow [12] and imagine that we gradually move the bulk black hole, initially at the center of AdS, towards the boundary brane.[4] As the bulk black hole approaches the brane, the dual view from the boundary is that a spherical 'blob' of thermal conformal fields is contracting. The maximally extended spacetime always contains a second asymptotic region, at whose boundary a CFT_2 is defined. This is entangled with the CFT_1 so the total state of the field is always pure.

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[4]This need not be an actual evolution in time, but may instead be regarded as a one-parameter sequence of eternal black hole spacetimes, in each of which the black hole is moving about the center of AdS with increasing amplitude of oscillation. In this way we can more easily work with the concept of two causally disconnected asymptotic regions.

When the black hole reaches the brane and sticks to it (possibly only temporarily), the dual description is that the thermal conformal field collapses and forms a black hole. This is the kind of black hole localized on the brane that we have been studying in the previous sections. Now, this black hole on the boundary brane can again be maximally extended across the horizon to another asymptotic region on the brane. It is in this second asymptotic region at the boundary where we find the CFT$_2$. But now notice that the two CFTs live in the presence of a black hole, one at each side of the horizon. The total state of the CFT is a pure, Hartle-Hawking vacuum, and tracing over CFT$_2$ we obtain a thermal state for the CFT$_1$ that lives in 'our' side of the horizon. This is exactly the same thermofield double in the presence of a black hole spacetime as discussed in [2].

Roughly speaking, as the black hole moves towards the brane, the CFT$_2$ is 'carried along' with it, so the CFT$_1$ and CFT$_2$ are gradually getting together. When the black hole reaches the brane, the two boundaries where the CFTs are defined become the two asymptotic regions of the brane section of the black hole.

In this manner we interpolate between the pictures in [10] and in this paper, of two entangled copies of the CFT. At all times the entropy of the black hole arises from this entanglement, the only difference being in whether the boundary where the CFT is defined contains the black hole horizon or not.

8. Discussion

The extreme simplicity of the argument for the main result in this paper, eq. (4.1), may prompt the criticism that the reasoning might actually be circular, assuming already what it purports to prove — an entropy equal to an area divided by $4G$. This is not the case. The holographic definition (2.1) is certainly inspired by the Bekenstein-Hawking formula, but it is actually different from it and is designed to compute a magnitude that is independent of the possible presence of a black hole (either in the bulk or the boundary) and which has a definite interpretation in the dual CFT theory even in the absence of gravity. To repeat it simply and precisely, our claim, not more nor less, is that the proposal of [9] for calculating the entanglement entropy of a field theory using its gravitational dual, (2.1), does imply that black hole entropy can be interpreted as entropy of entanglement.

The main strength of our derivation is its generality: it applies to any D-dimensional system of a black hole, and quantum fields in its presence, that admits a $(D+1)$-dimensional gravitational dual. Admittedly, the directness of the argument leading to (4.1) stems from the fact that all our calculations are done in the same side of the AdS/CFT duality — namely, the AdS side. This does not trivialize the conclusion, but it limits the insights we can obtain. In particular, we do not claim to have provided a microscopic counting of black hole entropy as entanglement entropy, although our work definitely supports the idea that such a calculation should in principle be possible. A microscopic counting would consist in the computation, within quantum field theory, of the entanglement entropy of the CFT across the horizon of a black hole that solves the equations (3.1). As such, this would be a test of the prescription (2.1). For deSitter space in $D = 2$, where the dual bulk solution is very simple and the divergences of the entanglement entropy are only logarithmic, this

calculation was carried out successfully in [12]. It seems very difficult, though, to extend these results beyond two-dimensional deSitter. For the case $D = 3$ analyzed in most detail here, besides the problems in controlling the divergences, our lack of knowledge about the theory on a stack of M2-branes dual to $\mathrm{AdS}_4(\times S^7)$, and more generally, the difficulties associated to field theory calculations at strong coupling, make a microscopic calculation including precise numerical factors unfeasible in practice.

Nevertheless, the AdS/CFT duality has been subject to extensive tests, and the formula (2.1) seems to hold its ground very well too, so one may be tempted to assume it and conclude from our results that it is indeed possible to view black hole entropy as entanglement entropy. At the very least, we think we have provided new grounds in support of this interpretation.

Acknowledgments

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